Two-Photon Exchange Contribution to the Electron-Neutron Elastic Scattering Cross Section and Data Calibrations for Gas Electron Multiplier Tracking Detectors

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nTPE and GEM Data Calibrations

Jefferson Lab

- 12 GeV Electron Accelerator capable of conducting experiments simultaneously in 4 different Halls, located in Newport News Virginia and run by the Department of Energy
- Performed in Hall A



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The Super BigBite Apparatus and Experiment Program



Calorimeter (HCal)

Electron Arm

BigBite Spectrometer including GEMs, GRINCH, Calorimeters, and Timing Hodoscope

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nTPE and GEM Data Calibrations

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Proton Form Factor Ratio and Proton Two-Photon Exchange (pTPE)



- Nucleon Form Factors (G_E, G_M) are fundamental observables describing the structure of the nucleon
- G_F/G_M as measured using cross-section data "Rosenbluth Separation" with a value of 1.0
- G_E/G_M as measured using polarization technique disagrees, especially at high Q^2 (3-4 sigma)
- Rosenbluth Separation is sensitive to TPE, while polarization technique is mostly insensitive. TPE could explain discrepancy
- Understanding TPE effects would provide a more complete characterization of G_F and G_M < □ > < 同 > < 回 > < 回 < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < < □ < □ < < □ < □ < < □ < < □ < □ < < □ < □ < □ < < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ < □ <

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Rosenbluth Separation for Nucleon Form Factors



Measurement of Rosenbluth Separation used to extract proton form factors

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \left(\frac{\alpha}{4MQ^2} \frac{E'}{E}\right)^2 |M_{\gamma}|^2 \\ &= \frac{\sigma_{Mott}}{\epsilon(1+\tau)} \left(\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right) \\ &= \frac{\sigma_{Mott}}{\epsilon(1+\tau)} \sigma_R \end{aligned}$$

- $\frac{d\sigma}{d\Omega}$ is the differential Born cross-section for electron-nucleon scattering, with invariant amplitude M_{γ} .
- α is the fine structure constant.
- σ_{Mott} is the scattering for a point-like particle.
- ϵ is the longitudinal polarization of the virtual photon.
- $\tau \equiv \frac{Q^2}{4M^2}$. *E* and *E'* are initial and final state energies.

Interference of OPE and TPE diagrams contribute to the cross-section measurement and allow for extraction of TPE affects



One Photon Exchange and Two-Photon Exchange Diagram

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- Contribution of TPE could reach 30% of the Rosenbluth Slope value at 5 $(GeV/c)^2$
- SBS nTPE experiment is first measurement of the Rosenbluth slope for the neutron using the ratio method
- Data taken in January & February 2022 for total of 19 days at $Q^2 = 4.5 \; ({\rm GeV/c})^2$ and 2 different ϵ values
- Results will be limited by systematics and not statistics

Gas Electron Multiplier (GEM) detectors

- GEMs are a type of gaseous ionization detector reliant on the concept of electron avalanche and part of the subclass of detectors known as Micro-Pattern Gaseous Detectors (MPGDs)
- Used for tracking detectors, preamplification, drift chambers, time projection chambers, and radiation imaging
- $\bullet\,$ Single detector gains are 10^3 or $10^4,$ depending on size and quality of GEM



Diagram of a typical GEM electrode





Diagram of a single GEM detector with Cartesian Readout

Electric Field in the region of the holes of the GEM electrode

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GEM Detectors for SBS Program





INFN XY-GEM Layer schematic and picture with RF shielding





UVA UV-GEM Layer schematic and picture with RF shielding



UVA XY-GEM Layer schematic and picture without RF shielding



- 4 INFN XY-GEM layers prepared for SBS program
- 4 UVA UV-GEM layers prepared for SBS program
- 11 UVA XY-GEM layers prepared for SBS program
- 2 INFN GEM layers operated during nTPE
- 2 UVA UV-GEM layers operated during nTPE
- 2 more UVA UV-GEM layers moved to BigBite during nTPE

UVA = University of Virginia, INFN = Istituto Nazionale Fisica Nucleare,

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GEM Data Calibrations



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Goal of Gain Match: Compare signal amplitudes from amplifier cards (APV). Correct amplitude variations for each amplifier card by generating gain coefficients. Applying gain coefficients should improve the GEM track-based efficiency and detector resolution. Compare 1 amplitude from different amplifier cards (sets of strips). For every event with a 'good' track, create a histogram for the ADC asymmetry $\left(\frac{ADC_{U/X} - ADC_{V/Y}}{ADC_{U/X} + ADC_{V/Y}}\right)$ between every U/X and V/Y APV combination on a GEM module.



Gain Match Pt 2



Generate ADC distributions for all hits and ADC distributions per module and apply Landua fit



Determine the Target ADC value from the Mean Peak Value (MPV).Target ADC value will be used to determine the reference amplifier card and for determining coefficients. Determine the APV in the U/X direction with the most statistics, use as a reference. Using a χ^2 minimization iteratively determine the relative iternal gain coefficients for each APV from the ADC asymmetries and the Target ADC value. Gain Coefficients vary from 0.5 to 2.0

Gain Coefficients vs APV



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Gain Match Pt 3



Pipeline coefficients and thresholds to replay to see affect on GEM signals, Track Based Efficiencies, and resolution. Initial Result: Track-Based efficiency improves by 2-4% from Gain Match

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Gain Match



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Takeaway:

nTPE Summary:

- First measurement of Rosenbluth slope on the neutron. Sensitive to TPE effects.
- Data analysis is ongoing, first-pass underway
- nTPE measurement provides further understanding about nucleon form factors

GEM Gain Match Future Work:

- $\bullet\,$ Finish first-pass creation of gain coefficients for GMn/nTPE data set
- Evaluate gain coefficients at different particle rates through the GEMs
- Evaluate gain coefficient stability over time
- Preliminary: Gain matching GEM signals increases Track-Based Efficiency by 2-4%

Acknowledgments

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- Graduate Students and Post-Docs
- Core Group of Shifters
- INFN GEM group
- UVA GEM group
- SBS Spokespeople

W&M Parity Group: Advisor: David Armstrong Graduate Students:

- Victoria Owen
- Ezekiel Wertz
- Kate Evans

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Thank You! Questions?



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GEM Electronics Readout





GEM Readout Electronics

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APV25 Card



APV25 Card, 128 Channels, 3.4 μ s trigger latency, capable of sampling signal at 40 MHz, 100 kHz readout rate

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INFN GEM Performance with Liquid Hydrogen (LH2) Target



Top: Two-Dimensional Cluster Map for layer J0 on LH2 at 1 $\mu \rm Amp$ beam current. Bottom: Track-Based Efficiencies on LH2 at 1 $\mu \rm Amp$ beam current

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RF Shielding and Pedestal Improvement



Example comparison of Common Mode baseline fluctuation, when in experimental Hall.

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laver with shielding.

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INFN GEM Linearity Studies



- Nonlinear detector high voltage is caused by divider scheme. The divider moves current away from the 3rd GEM foil causing a sag in gain.
- GEM particle rates were high during G_M^n run period.

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Improvements Made:

• RF (Aluminum) Shielding is important to reduce pedestal noise (common-mode corrections) and provide operable signal-noise detector conditions.

Future Improvements:

• To handle high particle rates the High Voltage divider will need to be designed for higher currents. This will reduce gain sagging effects.

Overall 5 out of the 6 INFN GEM modules had some sort of challenge during the G_M^n run period. So these occurrences need to be better understood for the remaining SBS program



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Start:



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Gain Match



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Config 3



XY Back tracker

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nTPE and GEM Data Calibrations

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- Big Picture: While TPE has been studied for the proton, there is essentially no TPE data for the neutron
- No free neutron targets

Start: $R_{n/p}$ is the ratio of quasi-elastic yields in scattering from a deuteron target. $N_{e,e'n}$ and $N_{e,e'p}$ are the quasi-elastic detector yields for neutrons and protons.

$$R_{n/p} \equiv R_{observed} = \frac{N_{e,e'n}}{N_{e,e'p}}$$

Apply corrections for hadron efficiencies, radiative corrections, final state effects, and re-scattering. Call this ratio $R_{corrected}$, its proportional to $\sigma_{L}^{n(p)}$ and $\sigma_{T}^{n(p)}$.

Now fix $Q^2 = 4.5$ (GeV/c)² and consider two different kinematic points (ϵ_1 and ϵ_2).

Take a corrected ratio for each kinematic point, call them $R_{corrected,\epsilon_1}$ and $R_{corrected,\epsilon_2}$.

Consider the ratio of the two corrected ratios and define $S_c^{n(p)} = \sigma_L^{n(p)} / \sigma_T^{n(p)}$

$$A = \frac{R_{corrected,\epsilon_1}}{R_{corrected,\epsilon_2}} = B \times \frac{1 + \epsilon_1 S_c^n}{1 + \epsilon_2 S_c^n} \approx B \times (1 + \Delta \epsilon \cdot S_c^n)$$

B only contains known proton information.

End: Two unknowns are σ_L^n and σ_T^n , which can be extracted.